Model Selection for Sequential Inference and Decision-making

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Learning & Adaptive Systems

Sequential Decision-Making & Bandits: Problem

At every step t

Choose actions $\boldsymbol{x}_t$

unknown reward

$$y_t = r(\boldsymbol{x}_t) + \varepsilon_t$$

obsv.noise

Receive feedback y_t

Repeat

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Goal: Choose actions that give a high reward

$$R(T) = \sum_{t=1}^{T} r(\boldsymbol{x}^\star) - r(\boldsymbol{x}_t)$$

image source: flaticon

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Motivation: maximize r using the fewest queries

image source: flaticon

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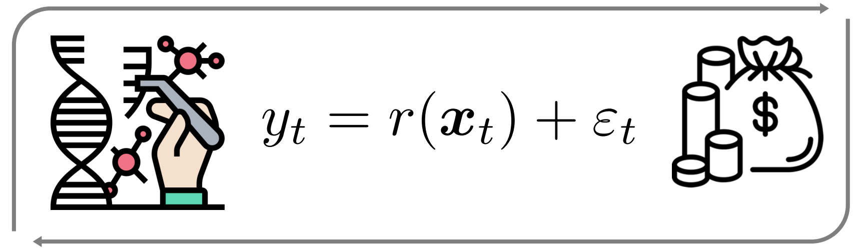

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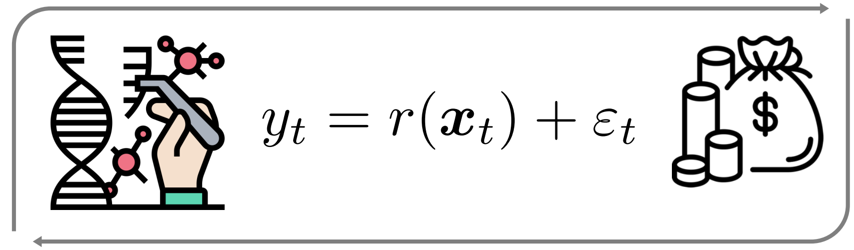

To take actions at every step:

- Estimate the reward function

based on:

- Statistical model for the reward e.g. r is a linear function
- history $H_{t-1} = \{(\boldsymbol{x}_1, y_1), \ldots, (\boldsymbol{x}_{t-1}, y_{t-1})\}$

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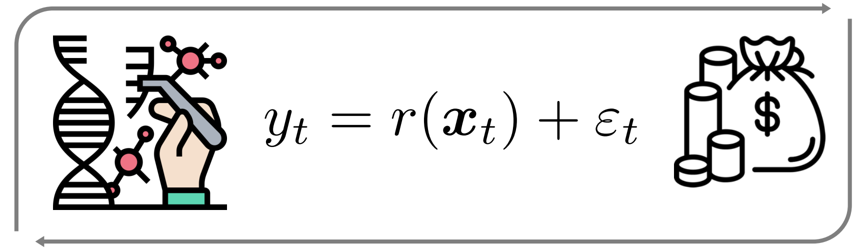

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Heavily rely on the choice of model → Model selection is key!

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Open problem: when is online model selection possible?

[Agarwal et al. 2017]

image source: flaticon

Online Model Selection problem

Find $j^\star$ while maximizing for the unknown r

$$R(T) = \sum_{t=1}^{T} r(\boldsymbol{x}^\star) - r(\boldsymbol{x}_t)$$

– Sublinear in T

– $\log M$

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Why do we need to select?
Why not just try out everything?

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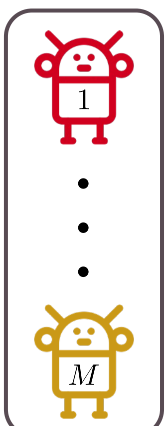
Instatiate M algorithms each using a different model



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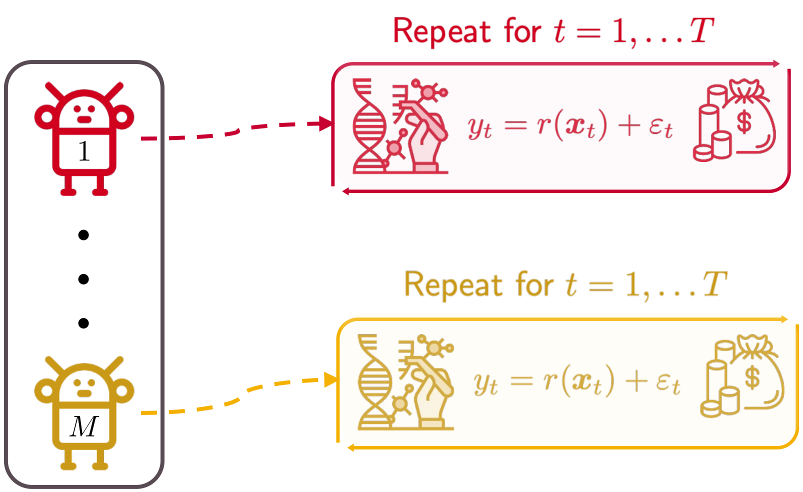

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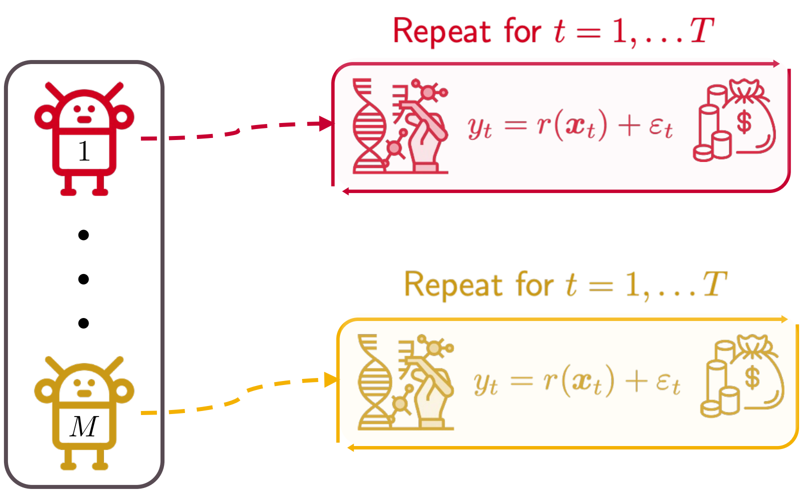

Statistically expensive
$\leftrightarrow$ High regret

$$\mathrm{poly}(M)$$

image source: flaticon

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The reward is linearly parametrized by an unknown feature map

Model Class $\{\boldsymbol{\phi}_j : \mathbb{R}^{d_0} \to \mathbb{R}^d,\ j = 1, \dots, M\}$ $\quad M \gg T$

$$\exists j^\star \in [M] \text{ s.t. } r(\cdot) = \boldsymbol{\theta}_{j^\star}^\top \boldsymbol{\phi}_{j^\star}(\cdot)$$

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Is not any-time: only works if horizon T is known in advance

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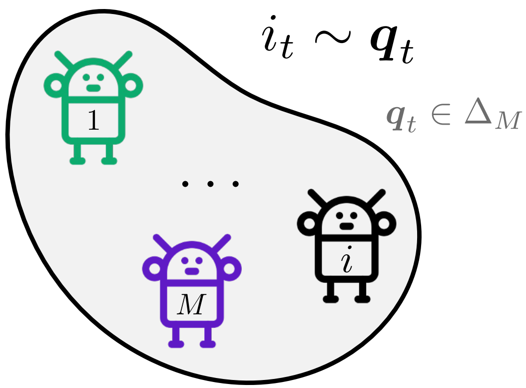

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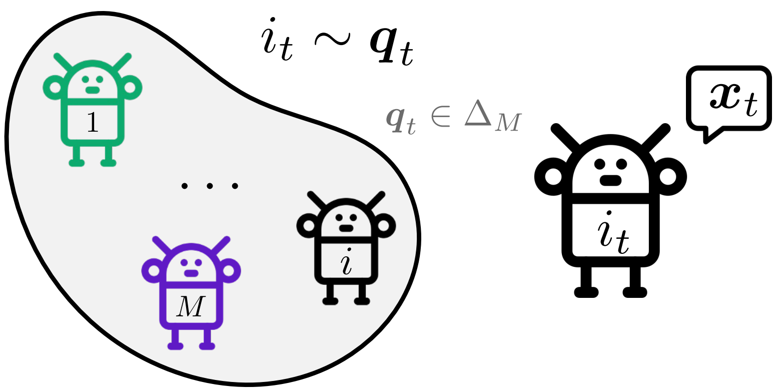

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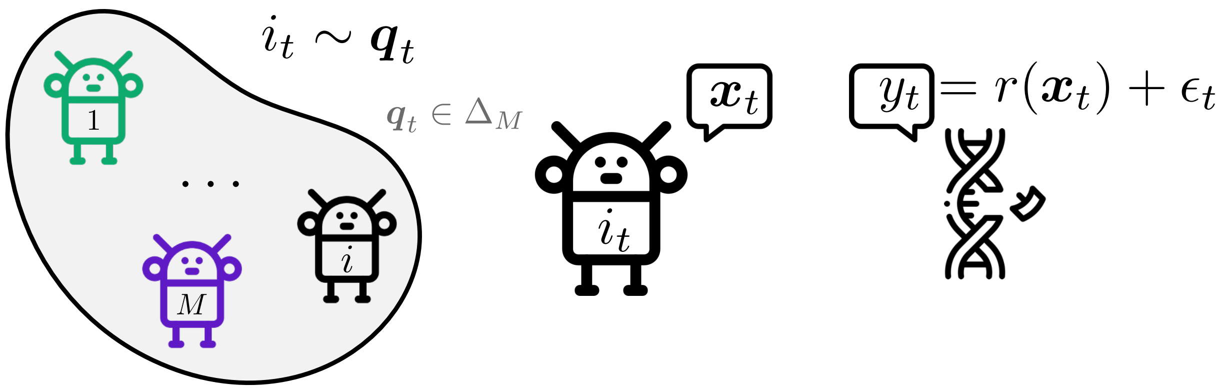

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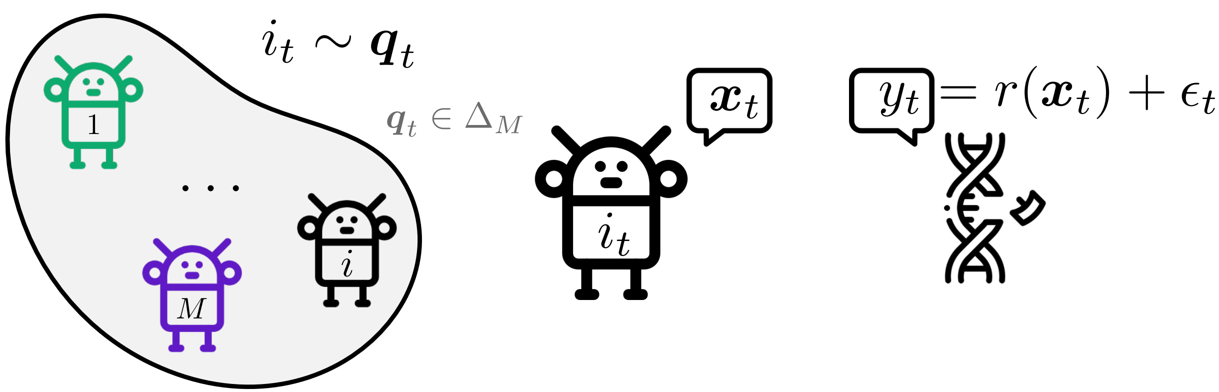

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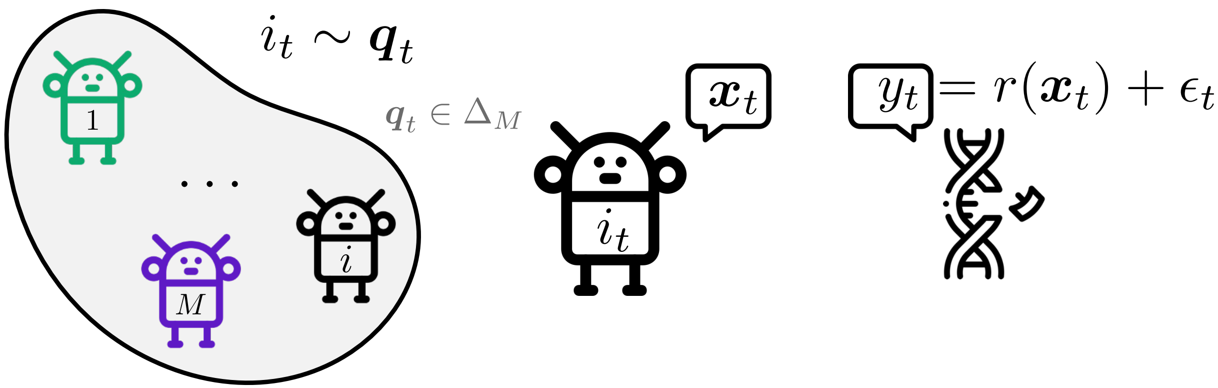

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Requires having observed the reward for the choice of each agent

Reward not observed? Hallucinate it.

image source: flaticon

How to hallucinate rewards

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Turn group lasso into a sparse online regression oracle

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Theorem (Anytime Lasso Conf Seq)

For appropriate choice of $(\lambda_t)_{t\geq 1}$,

$$\mathbb{P}\left(\forall t \geq 1 : \; \left\|\boldsymbol{\theta} - \hat{\boldsymbol{\theta}}_t\right\|_2 \lessapprox \sqrt{\frac{\log(M/\delta)}{t}}\right) \geq 1 - \delta$$

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Hallucinate the reward of agent j as

$$\hat{r}_{t,j} = \mathbb{E}_{\boldsymbol{x}\sim p_{t,j}} \hat{\boldsymbol{\theta}}_t^\top \boldsymbol{\phi}(\boldsymbol{x})$$

$p_{t,j} \in \mathcal{M}(\mathcal{X})$ action selection strategy

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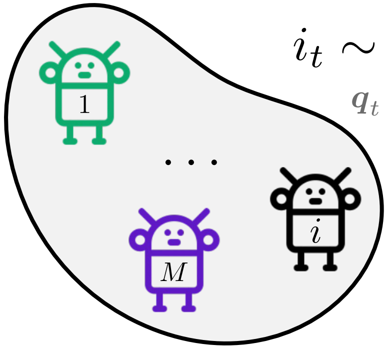

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increase $q_{t,j}$ if $\hat{r}_{t,j}$ was high

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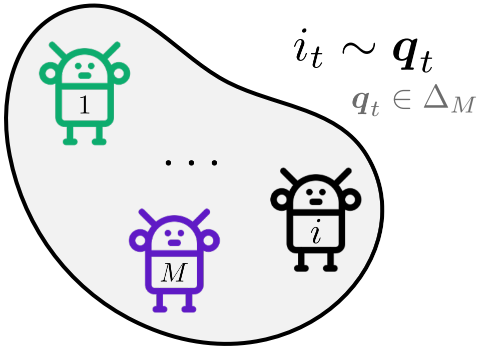

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$$q_{t,j} = \frac{\exp(\eta_t \sum_{s=1}^{t-1} \hat{r}_{s,j})}{\sum_{i=1}^{M} \exp(\eta_t \sum_{s=1}^{t-1} \hat{r}_{s,i})}$$

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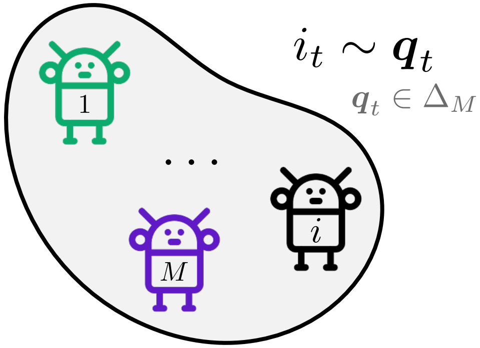

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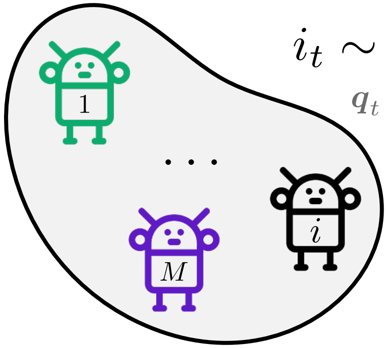

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sensitivity of updates

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Putting it all together

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Anytime **Exp**onential weighting algorithm with **L**asso reward estimates

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Anytime **Exp**onential weighting algorithm with **L**asso reward estimates

Algorithm 1 ALEXP

Inputs: γ_t, η_t, λ_t for $t \geq 1$

for $t \geq 1$ **do**

Draw $\boldsymbol{x}_t \sim (1-\gamma_t)\sum_{j=1}^{M} q_{t,j} p_{t,j} + \gamma_t \text{Unif}(\mathcal{X})$

Observe $y_t = r(\boldsymbol{x}_t) + \epsilon_t$.

Append history $H_t = H_{t-1} \cup \{(\boldsymbol{x}_t, y_t)\}$.

Update agents $p_{t,j}$ for $j = 1, \dots, M$.

Calculate $\hat{\boldsymbol{\theta}}_t \leftarrow \text{Lasso}(H_t, \lambda_t)$ and estimate

$$\hat{r}_{t,j} \leftarrow \mathbb{E}_{\boldsymbol{x}\sim p_{t+1,j}}[\hat{\boldsymbol{\theta}}_t^\top \boldsymbol{\phi}(\boldsymbol{x})]$$

Update selection distribution

$$q_{t+1,j} \leftarrow \frac{\exp(\eta_t \sum_{s=1}^{t} \hat{r}_{s,j})}{\sum_{i=1}^{M} \exp(\eta_t \sum_{s=1}^{t} \hat{r}_{s,i})}$$

Putting it all together

Find $j^\star$ while maximizing for the unknown r

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Putting it all together: ALExp

Find $j^\star$ while maximizing for the unknown r

Anytime **Exp**onential weighting algorithm with **L**asso reward estimates

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Theorem (Online Model Selection)

For appropriate choices of parameters, ALEXP satisfies

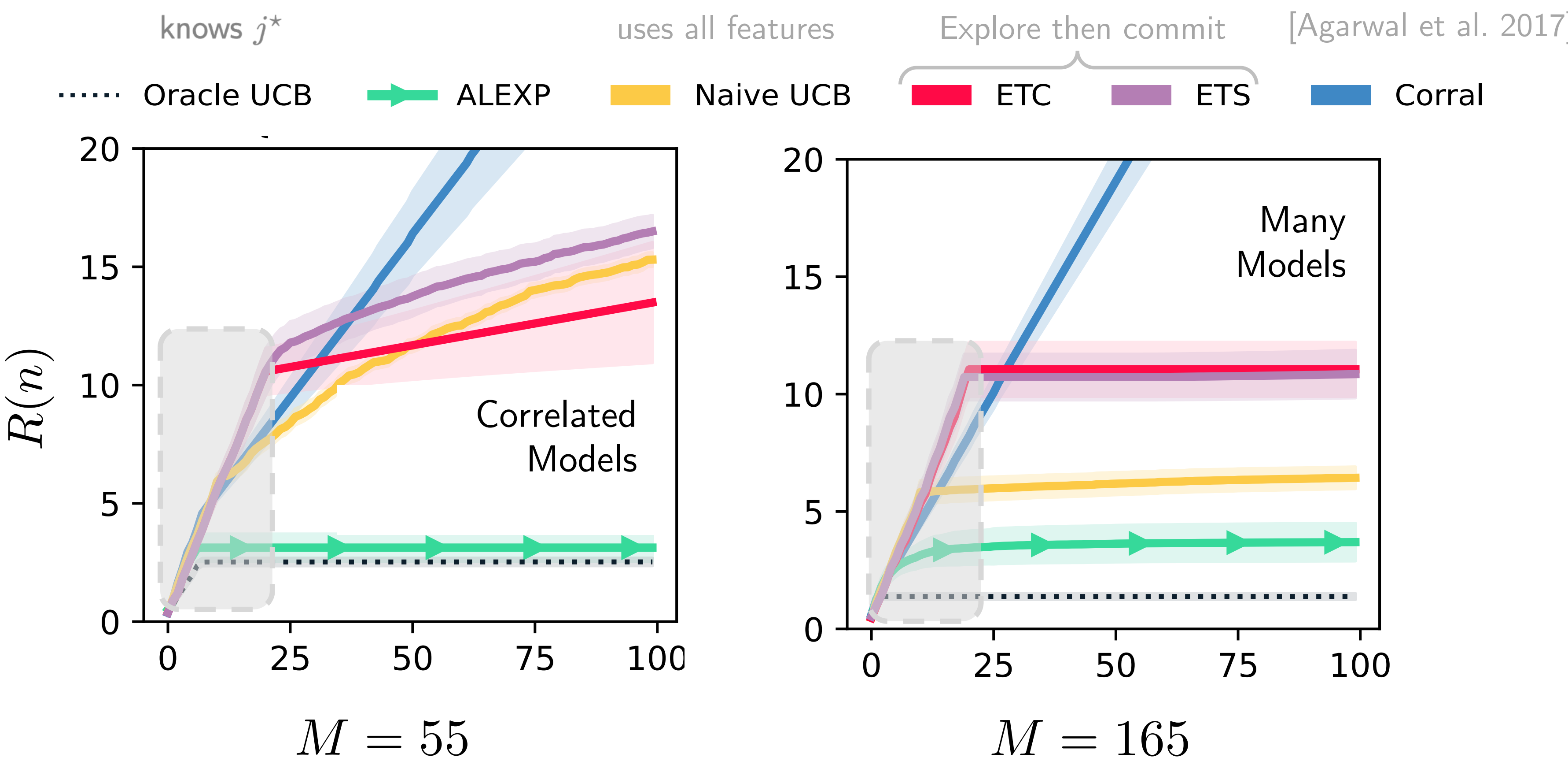
$$R(T) = \mathcal{O}\left(\sqrt{T \log^3 M} + T^{3/4}\sqrt{\log M}\right)$$

w.h.p. simultaneously for all $T \geq 1$.

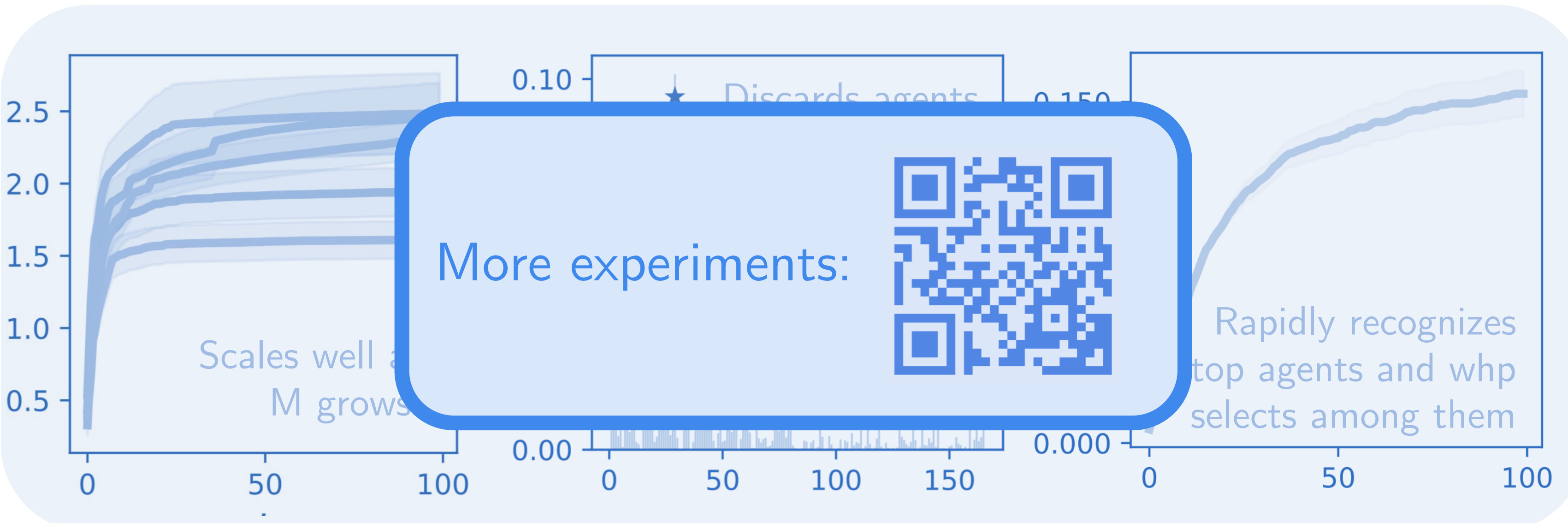
[Solves the open problem of Agarwal et al. 2017 in the Linear case]

Model Selection for Optimistic algorithms

data generation & baselines
described in the paper.



If I am running out of time:



If not...

Model Selection Dynamics of ALExp

Let's see how things evolve turing training...

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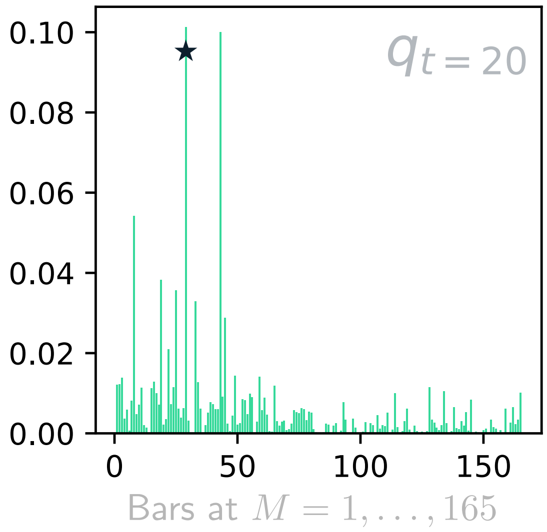
Distribution over the models
at time t=20



Model Selection Dynamics of ALExp

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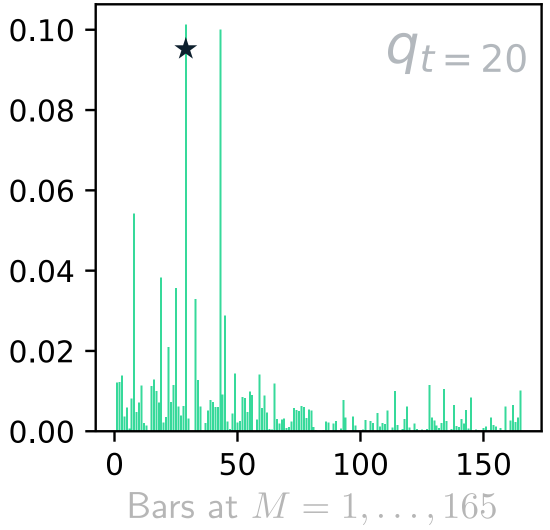

Discards agents without
having queried them

Model Selection Dynamics of ALExp

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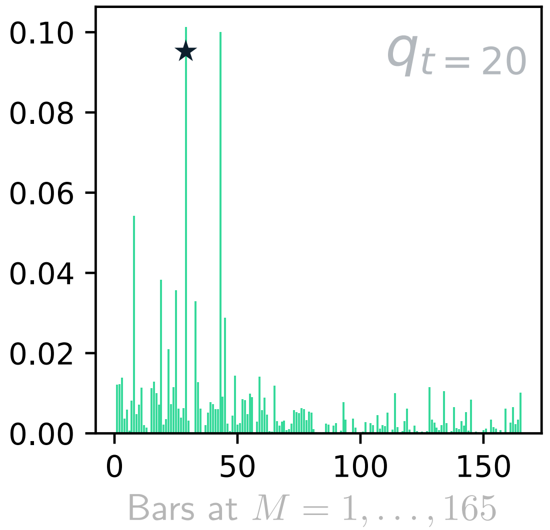
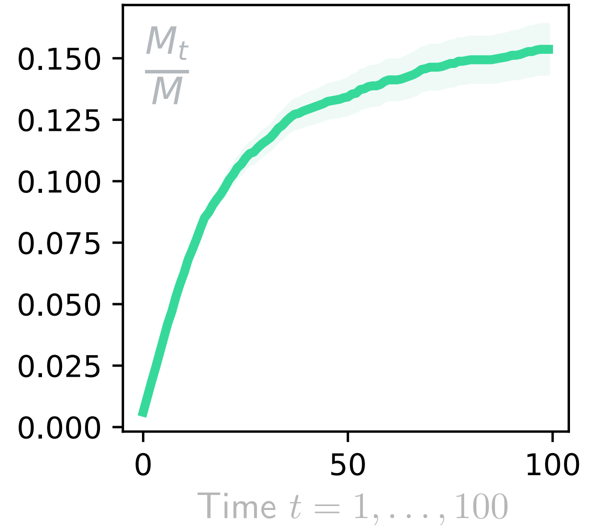


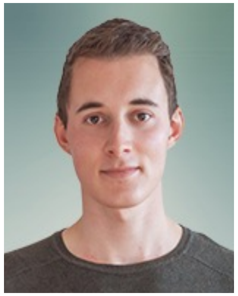
Discards agents without
having queried them

Number of visited agents
Total number of agents



Rapidly recognizes top agents
and whp selects among them

Thank you!



PK, Nicolas Emmenegger, AK, and Aldo Pacchiano. "Anytime Model Selection in Linear Bandits." NeurIPS, 2023.

PK, Jonas Rothfuss, and AK. "Meta-learning hypothesis spaces for sequential decision-making." ICML, 2022.

Schur, Felix, **PK**, Jonas Rothfuss, and AK. "Lifelong bandit optimization: no prior and no regret." UAI, 2023.

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Theorem (Regret - Informal)

For appropriate choices of $(\gamma_t, \lambda_t, \eta_t)$,

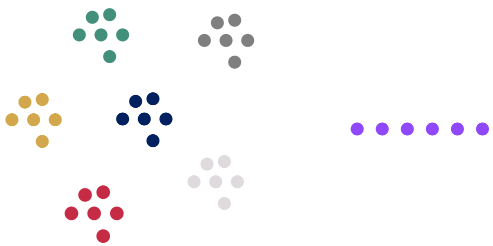
$$R(n) = \mathcal{O}\left(C(M,\delta,d)\left(\sqrt{n}\log M + n^{3/4}\right)\right)$$

with probability greater than $1-\delta$*,*
simultaneously for all $n \geq 1$.

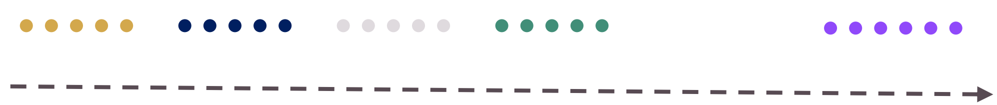
$$C(M,\delta,d) = \mathcal{O}\Big(\sqrt{d\log M/\delta + \sqrt{d\log M/\delta}}\Big)$$

We consider 3 scenarios of increasing diffculty

1. Offline Data from similar tasks is available [KRK 2022]



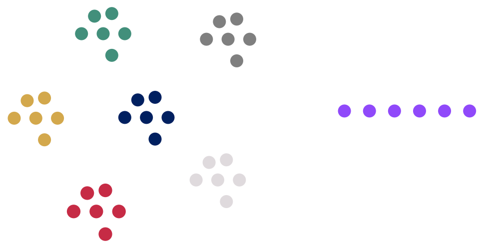
2. Online data from similar tasks can be available [SKRK 2023]



3. No data from similar tasks is available [KPEK 2023]

Meta-Model Selection: Offline

When offline data from similar tasks is available,



$$y_{s,i} = r_s(\boldsymbol{x}_{s,i}) + \varepsilon_{s,i} \qquad i = 1, \dots, n \text{ and } s = 1, \dots, m$$

$$r_s(\cdot) = \sum_{j=1}^{M} \langle \boldsymbol{\theta}_s^{(j)}, \boldsymbol{\phi}_j(\cdot) \rangle \qquad J \text{ is shared}$$

Classical feature selection with Lasso

$$\hat{\boldsymbol{\theta}}^{(1)}, \dots, \hat{\boldsymbol{\theta}}^{(M)} = \arg\min \frac{1}{mn} \|\boldsymbol{y} - \sum_{j=1}^{M} \Phi_j \boldsymbol{\theta}^{(j)}\|_2^2 + \lambda \sum_{j=1}^{M} ||\boldsymbol{\theta}^{(j)}||_2$$

$$\hat{J} = \{j \in [M] \text{ s.t. } \hat{\boldsymbol{\theta}}^{(j)} > \omega\}$$

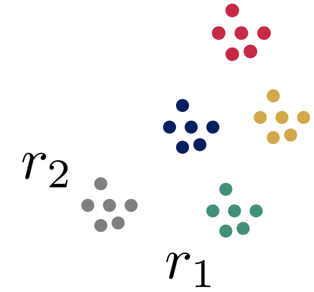
Solving the online optimization problem using the learned model

Meta Model Selection: Lifelong

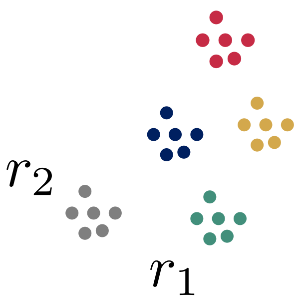
$\forall s \geq 1 : r_s \in \mathcal{H}$

Suppose the bandit task is of repetitive nature,

Optimizing for different molecular properties
Recommending products to different costumers



Updated model



Solve classic model selection problem

Solve new task

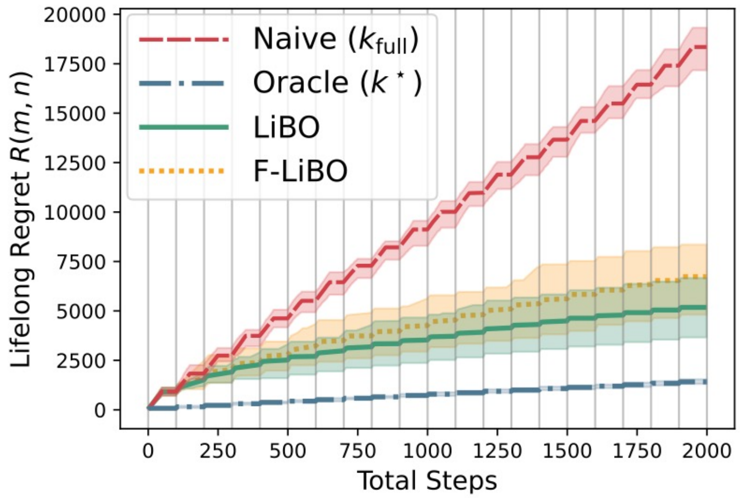
New dataset $(\boldsymbol{x}_{s,t}, y_{s,t})_{t \leq T}$

Theorem (Lifelong Model Selection)

Under mild assumptions on the meta-data, and for an appropriate choice of λ, w.h.p.

- *$\hat{J}$ is a consistent estimator of J,*
- *The optimization algorithm which uses $\hat{J}$ achieves oracle performance $R^\star(T, m)$,*

as m grows.



the regret converges at a $\mathcal{O}(\log M/\sqrt{m})$ rate

$$R(T, m) = \sum_{s=1}^{m} \sum_{t=1}^{T} r_s(\boldsymbol{x}_s^\star) - r_s(\boldsymbol{x}_{s,t})$$